ProjectManagement: an R package for managing projects

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Outline

1. Project Management
   - Deterministic Projects
   - Resource Management
   - Delay Costs Allocation
2. Stochastic Projects
   - Stochastic Project Management
   - Delay Costs Allocation

A Project

Formally, a project is a 3-tuple $(N, \prec, x^0)$ where:

- N is the finite set of activities.
- $\prec$ is a binary relation over N satisfying asymmetry and transitivity. For every $i, j \in N$, we interpret $i \prec j$ as "activity j cannot start until activity i has finished".
- $x^0 \in \mathbb{R}^N$ is the vector of estimated durations. For every $i \in N$, x_i^0 is a non-negative real number indicating the estimated duration of activity i.

An Example

N	1	2	3	4	5
Immediate precedence	-	-	-	2	3
Durations	2	1.5	1	1.5	2

N	6	7	8	9	10
Immediate precedence	3	1, 4	2	5, 8	6
Durations	2.5	3	4	2	5

Table: Example of a deterministic project.

An Example

Example

```
> prec<-matrix(0,nrow=10,ncol=10)
> prec[1,7]<-1; prec[2,4]<-1; prec[2,8]<-1;
> prec[3,5]<-1; prec[3,6]<-1; prec[4,7]<-1;
> prec[5,9]<-1; prec[6,10]<-1; prec[8,9]<-1;
```

Example

```
> dag.plot(prec)
```

An Example

Example

```
> prec<-matrix(0,nrow=10,ncol=10)
> prec[1,7]<-1; prec[2,4]<-1; prec[2,8]<-1;
> prec[3,5]<-1; prec[3,6]<-1; prec[4,7]<-1;
> prec[5,9]<-1; prec[6,10]<-1; prec[8,9]<-1;
```

Example

```
> dag.plot(prec)
```

An Example

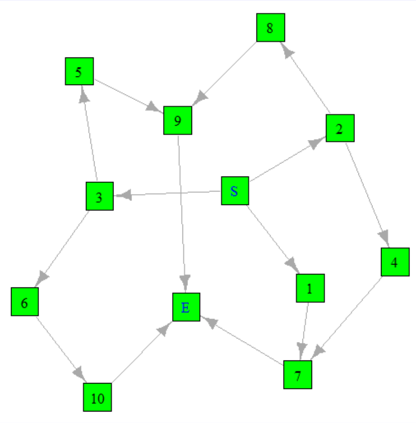

Figure: AON graph of the project.

An Example

Example

```
> duration<-c(2,1.5,1,1.5,2,2.5,3,4,2,5)
```

Example

```
> schedule.pert(duration,prec)
```

An Example

Example

```
> duration<-c(2,1.5,1,1.5,2,2.5,3,4,2,5)
```

Example

```
> schedule.pert(duration,prec)
```

An Example

Example

'Total duration of the project'
[1] 8.5

	Duration	Earliest start	Latest start	Earliest completion
1	2.0	0.0	3.5	2.0
2	1.5	0.0	1.0	1.5
3	1.0	0.0	0.0	1.0
4	1.5	1.5	4.0	3.0
5	2.0	1.0	4.5	3.0
6	2.5	1.0	1.0	3.5
7	3.0	3.0	5.5	6.0
8	4.0	1.5	2.5	5.5
9	2.0	5.5	6.5	7.5
10	5.0	3.5	3.5	8.5

An Example

Example

	Latest completion	Slack	Free Slack	Independent Slack
1	5.5	3.5	1.0	0.0
2	2.5	1.0	0.0	0.0
3	1.0	0.0	0.0	0.0
4	5.5	2.5	0.0	0.0
5	6.5	3.5	2.5	0.0
6	3.5	0.0	0.0	0.0
7	8.5	2.5	2.5	0.0
8	6.5	1.0	0.0	0.0
9	8.5	1.0	1.0	0.0
10	8.5	0.0	0.0	0.0

An Example

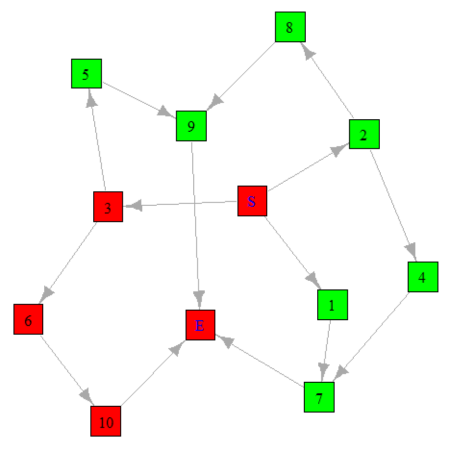

Figure: AON graph of the project. Nodes in red indicate critical activities

Resource Management

- The minimal cost expediting considers that the duration of some activities can be reduced by increasing the resources allocated to them and thus the implementation costs.
- Levelling of resources: execute the project in its minimum duration time whilst the use of resources is as uniform as possible over time.
- Allocation of resources: the level of resources available in each period of time is limited.

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An Example

Example

```
> minimum.durations<-c(1,1,0.5,1,1,2,2,3,1,3)
> activities.costs<-c(1,2,1,1,3,2,1,2,3,5)
```

Example

```
> mce(duration,minimum.durations,prec,
activities.costs,duration.project=NULL)
```

An Example

Example

```
> minimum.durations<-c(1,1,0.5,1,1,2,2,3,1,3)
> activities.costs<-c(1,2,1,1,3,2,1,2,3,5)
```

Example

```
> mce(duration,minimum.durations,prec,
activities.costs,duration.project=NULL)
```

An Example

Example

necessary negative increase

1: 0.5

Read 1 item

Project duration =

[1] 8.0 7.5 7.0 6.5 6.0 5.5

An Example

Example

necessary negative increase
1: 0.5
Read 1 item
Project duration =
[1] 8.0 7.5 7.0 6.5 6.0 5.5

An Example

Example

Estimated durations =

2.0	2.0	2.0	2.0	2.0	2.0
1.5	1.5	1.5	1.5	1.0	1.0
0.5	0.5	0.5	0.5	0.5	0.5
1.5	1.5	1.5	1.5	1.5	1.5
2.0	2.0	2.0	2.0	2.0	2.0
2.5	2.0	2.0	2.0	2.0	2.0
3.0	3.0	3.0	3.0	3.0	3.0
4.0	4.0	3.5	3.0	3.0	3.0
2.0	2.0	2.0	2.0	2.0	1.5
5.0	5.0	4.5	4.0	3.5	3.0

Costs per solution =

0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	1.0	1.0
0.5	0.5	0.5	0.5	0.5	0.5
0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	0.0
0.0	1.0	1.0	1.0	1.0	1.0
0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	1.0	2.0	2.0	2.0
0.0	0.0	0.0	0.0	0.0	1.5
0.0	0.0	2.5	5.0	7.5	10.0

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An Example

Example

```
> resources<-c(2,3,4,3,3,4,2,2,5,2)
```

Example

```
> levelling.resources(duration,prec,resources,int=0.5)
Earliest start times =
[1] 0.0 0.5 0.0 2.0 4.5 1.0 3.5 2.5 6.5 3.5
Resources by period=
[1] 6 9 9 9 7 9 9 6 6 9 9 9 9 7 7 7 7
```

An Example

Example

```
> resources<-c(2,3,4,3,3,4,2,2,5,2)
```

Example

```
> levelling.resources(duration,prec,resources,int=0.5)
Earliest start times =
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An Example

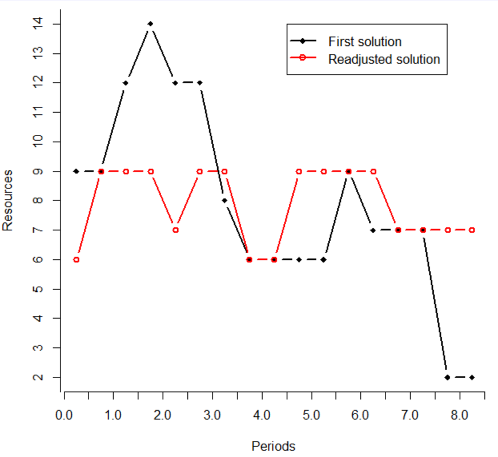

Figure: Levelling resources.

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An Example

Example

```
> max.resources<-8
```

Example

```
> resource.allocation(duration,prec,resources,
max.resources,int=0.5)
Project duration =
[1] 10
Earliest start times =
[1] 1.5 0.0 0.0 3.5 5.5 1.0 5.0 1.5 8.0 3.5
Resources by period =
[1] 7 7 7 8 8 8 8 7 7 7 6 7 7 7 7 4 7 5 5 5
```

An Example

Example

```
> max.resources<-8
```

Example

```
> resource.allocation(duration,prec,resources,
max.resources,int=0.5)
Project duration =
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Resources by period =
[1] 7 7 7 8 8 8 8 7 7 7 6 7 7 7 7 4 7 5 5 5
```

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A project with delays

A project with delays P is a tuple $(N, \prec, x^0, x, C)$ where:

- N is the finite set of activities.
- $\prec$ is a binary relation over N satisfying asymmetry and transitivity.
- $x^0 \in \mathbb{R}^N$ is the vector of estimated durations. For every $i \in N$, x_i^0 is a non-negative real number indicating the estimated duration of activity i.
- $x \in \mathbb{R}^N$ is the vector of actual durations. For every $i \in N$, $x_i \geq x_i^0$ indicates the actual duration of activity i.
- $C : \mathbb{R} \to \mathbb{R}$ is the delay cost function. We assume that C is non-decreasing and that $C(D(N, \prec, x^0)) = 0$.

Rules for Projects with Delays

The Proportional rule for projects with delays is defined by

$$\varphi_i(P) = \frac{x_i - x_i^0}{\sum_{j\in N} x_i - x_i^0} C(D(N, \prec, x))$$

for all $i \in N$.

The Shapley rule for projects with delays Sh is defined by $Sh(P) = \Phi(c^P)$

- c^P is the TU-game with set of players N given by $c^P(S) = C(D(N, \prec, (x_S, x^0_{N\setminus S})))$, for all $S \subset N$, and
- $\Phi(c^P)$ denotes the proposal of the Shapley value for c^P.

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An Example

Example

```
> observed.duration<-c(8,3,2,5,2,6,4,6,4,5.5)
> cost.function<-function(x)return(max(x-8.5,0))
```

Example

```
>delay.pert(duration,prec,observed.duration,delta=NULL,
cost.function)
There has been a delay of = 5
```

An Example

Example

```
> observed.duration<-c(8,3,2,5,2,6,4,6,4,5.5)
> cost.function<-function(x)return(max(x-8.5,0))
```

Example

```
>delay.pert(duration,prec,observed.duration,delta=NULL,
cost.function)
There has been a delay of = 5
```

An Example

Example

	1	2	3	4	5
The proportional payment	1.43	0.36	0.24	0.83	0.00
The truncated proportional payment	1.25	0.38	0.25	0.88	0.00
Shapley rule	0.71	0.40	0.55	0.26	1.68

	6	7	8	9	10
The proportional payment	0.83	0.24	0.48	0.48	0.12
The truncated proportional payment	0.88	0.25	0.5	0.5	0.13
Shapley rule	1.68	0.19	0.45	0.45	0.32

A Stochastic Project

Formally, a stochastic project is a 3-tuple $(N, \prec, X^0)$ where:

- N is the finite set of activities.
- $\prec$ is a binary relation over N satisfying asymmetry and transitivity. For every $i, j \in N$, we interpret $i \prec j$ as "activity j cannot start until activity i has finished".
- $X^0 \in \mathbb{R}^N$ is the vector of random durations. For every $i \in N$, X_i^0 is a non-negative random variable describing the duration of activity i.

An Example

Example

$$X^0 = (t(1,2,3), \exp(2/3), t(1/2,5/4,5/4), t(1/4,7/4,5/2), t(1,2,3)$$
$$t(1,3/2,5), t(1,1,7), t(3,4,5), t(1/2,5/2,3), t(1,6,8)),$$

where $t(a,b,c)$ denotes the triangular distribution with parameters a, b, and c, and $\exp(\alpha)$ denotes the exponential distribution with parameter α.

An Example

Example

```
> distribution<-c("TRIANGLE","EXPONENTIAL",rep("TRIANGLE",8))
> values<-
matrix(c(1,3,2,2/3,0,0,1/2,5/4,5/4,1/4,5/2,7/4,1,3,2,1,5,3/2,1,7,1,3,5,4,
1/2,3,5/2,1,8,6),nrow=10,ncol=3,byrow=T)

>stochastic.pert(prec,distribution,values,
percentile=0.95,plot.activities.times=c(7))

Average time of the project = 9.070575
Percentile duration of the project = 11.66658
Criticality index by activity 1.3 34.2 64.5 8.2 0 64.5 9.5 26 26 64.5
```

An Example

Example

```
> distribution<-c("TRIANGLE","EXPONENTIAL",rep("TRIANGLE",8))
> values<-
matrix(c(1,3,2,2/3,0,0,1/2,5/4,5/4,1/4,5/2,7/4,1,3,2,1,5,3/2,1,7,1,3,5,4,
1/2,3,5/2,1,8,6),nrow=10,ncol=3,byrow=T)

>stochastic.pert(prec,distribution,values,
percentile=0.95,plot.activities.times=c(7))

Average time of the project = 9.070575
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An Example

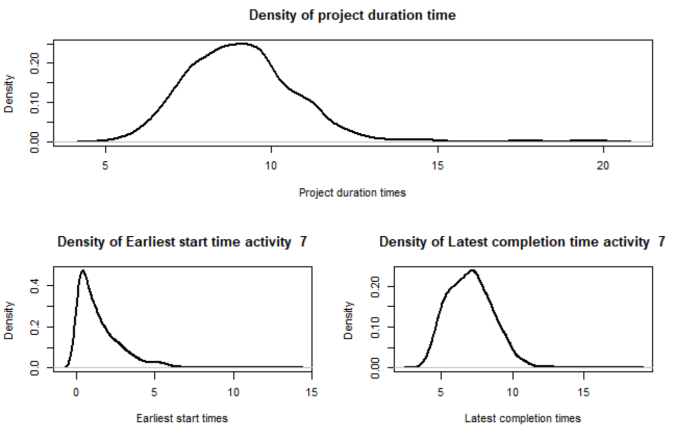

Figure: Density estimation of project duration time and earliest start and latest completion times for activities 7.

A stochastic project with delays

A stochastic project with delays SP is a tuple $(N, \prec, X^0, x, C)$ where:

- N is the finite set of activities.
- $\prec$ is a binary relation over N satisfying asymmetry and transitivity.
- $X^0 \in \mathbb{R}^N$ is the vector random durations. For every $i \in N$, x_i^0 is a non-negative random variable describing the duration of activity i.
- $x \in \mathbb{R}^N$ is the vector of actual durations. For every $i \in N$, $x_i \geq x_i^0$ indicates the actual duration of activity i.
- $C : \mathbb{R} \to \mathbb{R}$ is the delay cost function. We assume that C is non-decreasing and that $C(D(N, \prec, 0)) = 0$.

Rules for Projects with Delays

The Proportional rule for projects with delays is defined by

$$\varphi_i(SP) = \frac{x_i - \mathbb{E}(X_i^0)}{\sum_{j \in N} x_i - \mathbb{E}(X_j^0)} C(D(N, \prec, x))$$

for all $i \in N$.

The Shapley rule for projects with delays Sh is defined by $Sh(SP) = \Phi(c^{SP})$

- c^{SP} is the TU-game with set of players N given by $c^{SP}(S) = E(C(D(N, \prec, (x_S, X^0_{N \setminus S}))))$, for all $S \subset N$, and
- $\Phi(c^{SP})$ denotes the proposal of the Shapley value for c^{SP}.

Rules for Projects with Delays

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The Shapley rule for projects with delays Sh is defined by $Sh(SP) = \Phi(c^{SP})$

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- $\Phi(c^{SP})$ denotes the proposal of the Shapley value for c^{SP}.

An Example

Example

> delay.stochastic.pert(prec,distribution,values,
observed.duration,percentile=NULL,delta=NULL,
cost.function,compilations=1000)

Total delay of the stochastic project = 5

An Example

Example

	1	2	3	4	5
Proportional rule	1.43	0.36	0.24	0.84	0.00
Truncated proportional rule	1.25	0.38	0.25	0.88	0.00
Shapley rule	0.59	0.37	0.61	0.36	0.17
Shapley rule 2	0.49	0.42	0.64	0.29	0.07

	6	7	8	9	10
Proportional rule	0.83	0.24	0.48	0.48	0.12
Truncated proportional rule	0.88	0.25	0.5	0.5	0.13
Shapley rule	1.36	0.24	0.51	0.50	0.31
Shapley rule 2	1.52	0.18	0.48	0.46	0.47

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